Exam Symmetry in Physics

Date	April 2, 2013
Room	A Jacobshal 01
Time	9:00 - 12:00
Lecturer	D. Boer

- Write your name and student number on every separate sheet of paper
- You are not allowed to use the lecture notes, nor other notes or books
- All subquestions (a, b, etc) of the 3 exercises (18 in total) have equal weight
- Answers may be given in Dutch
- Illegible handwriting will be graded as incorrect
- Good luck!

Exercise 1

(a) Prove that the inverses of elements of a conjugacy class of a group G also form a conjugacy class of G.

(b) Show that the mapping $\phi : G \to G, g \mapsto g^{-1}$ is a 1-1 mapping that in general does not provide an isomorphism. Show for which type of groups it does provide an isomorphism.

(c) Consider a conjugacy class K. Show by using Schur's lemma that $O = \sum_{g \in K} D(g)$ is proportional to the identity element, if D is an irrep.

(d) Consider a regular n-sided polygon. Show that any rotation of the polygon is conjugated to its inverse rotation. Show this by using the defining relations of the group D_n and by geometrical arguments.

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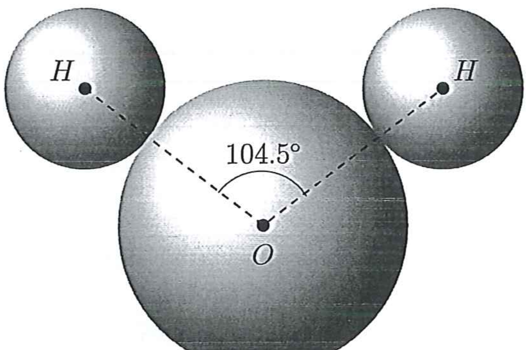
(e) Consider $R \in O(3)$. Compute the determinant of the inverse of R.

Exercise 2
Consider the water molecule H_2O:



(a) Determine the group G_W of all symmetry transformations that leave the water molecule invariant. (Hint: consider rotations *and* reflections in three dimensions.)

(b) Construct the character table of this group G_W.

(c) Construct the three-dimensional vector representation D^V of G_W.

(d) Decompose D^V into irreps of G_W and use this to conclude whether the water molecule allows for an electric dipole moment or not.

(e) Determine the Clebsch-Gordan series of the direct product representation $D^V \otimes D^V$ of G_W.

(f) Explicitly determine the tensors T^{ij} $(i, j = 1, 2, 3)$ that are invariant under the transformations of G_W and check whether the answer is in agreement with the result obtained in part (e) of this exercise.

Exercise 3

Consider the group of rotations in two dimensions $SO(2)$ and the unitary group $U(1)$.

(a) Write down the elements of $SO(2)$ in its defining representation.

(b) Write down the elements of $U(1)$ in its defining representation.

(c) Show that $SO(2) \cong U(1)$.

(d) Write down all (complex) irreducible representations of $SO(2)$.

(e) Give an example of a physical system with an $SO(2)$ or $U(1)$ symmetry.

Next consider the extension of $SO(2)$ to include reflections: the group $O(2)$ of orthogonal 2×2 matrices.

(f) Write down the two-dimensional representation of $O(2)$ obtained by its action on the vector

$$\begin{pmatrix} x + iy \\ x - iy \end{pmatrix}$$

(g) Show whether this two-dimensional rep of $O(2)$ is an irrep or not.